Technical Report on difference between maximum likelihood estimation vs marginal likelihood estimation

This technical report provides a comprehensive insight into comparison of maximum likelihood estimation and marginal likelihood estimation comparing core principle, objective mathematical formula, advantages and limitations, etc. Maximum likelihood estimation and marginal likelihood estimation are two commonly used statistical estimation approaches. These approaches might sound similar; however, the underlying principles and applications differ significantly.

1. **Fundamental Definitions**

**Maximum Likelihood Estimation**

* **Core principle:** Maximum likelihood estimation is a method that estimates parameters by maximizing the likelihood function of observed data
* **Objective:** Find parameter values that make the observed data most probable.
* **Mathematical formulation:** θ̂ = argmax[θ] L(θ | X) = argmax[θ] P(X | θ) Where θ represents model parameters and X represents observed data.

**Marginal Likelihood Estimation**

* **Core principle:** Marginal likelihood estimation is used primarily in Bayesian inference which integrates out the inconvenience parameters to estimate model evidence.
* **Objective:** Compute probability of data averaged over all possible parameters’ values.
* **Mathematical formulation:** P(X) = ∫ P(X | θ) P(θ) dθ, where P(θ) represents prior distribution of parameters.

1. **Key Differences**

**Computational Approach**

**Maximum Likelihood Estimation**

* Focuses on point estimates
* Maximizes likelihood for specific parameter values
* Does not explicitly account for parameter uncertainty
* Computationally simpler

**Marginal Likelihood Estimation**

* Integrates over entire parameter space
* Accounts for parameter uncertainty through priors
* Provides probabilistic model comparison
* Computationally more complex

1. **Practical Applications**

**Maximum Likelihood Estimation typical use cases**

* Parameter estimation in frequentist statistical frameworks
* Simple models with clear parameter relationships
* Scenarios with large, well-behaved datasets
* Machine learning classification and regression problems

**Marginal Likelihood Estimation typical use cases**

* Bayesian inference
* Complex models with multiple parameters
* Limited data scenarios
* Model comparison and selection
* Machine learning with hierarchical models

1. **Advantages and Limitations**

**Maximum Likelihood Estimation Advantages**

* Computationally efficient
* Provides clear point estimates
* Works well with large, informative datasets

**Maximum Likelihood Estimation Limitation**

* Assumes known parameter distribution
* Sensitive to outliers
* Limited uncertainty quantification

**Marginal Likelihood Estimation Advantages**

* Robust uncertainty estimation
* Incorporates prior knowledge
* Enables probabilistic reasoning

**Marginal Likelihood Estimation Limitations**

* Computationally intensive
* Requires specifying prior distributions
* More complex implementation

Although the underlying principles of Maximum Likelihood Estimation and Marginal Likelihood Estimation are slightly comparable, their applications, methods of calculation, and targets are not. While marginal likelihood estimation, which is frequently employed in Bayesian contexts, emphasizes on integrating out latent variables to determine the overall likelihood in a probabilistic model, maximum likelihood estimation is a frequentist method that maximizes the likelihood of data to estimate model parameters. The modeling framework (frequentist vs. Bayesian), the complexity of the data, and the model structure all play a significant role in determining whether to use each technique.